Mesh Parameterization for an Open Connected Surface without Partition

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Abstract

A novel mesh parametrization method for an open connected surface is presented. The parametrization method is based on Hessian-based Locally Linear Embedding (HLLE). Our method operates directly on the surface without using any partition technique and can preserve the local and global structure, while partition-based methods often produce high distortion and discontinuity nearby partition boundaries. In addition, some examples about texture mapping show the efficiency of our method

1. Introduction

Surface parametrization is a fundamental problem in computer graphics. It is essential for some operations such as texture mapping, mesh resampling, and mesh compression. A parameterization defines a mapping between regions on the 2D plane and the surface embedded in the 3D space [6]. This operation is called surface parameterization or surface flattening and this planar representation is called embedding. There have been many surface parameterization methods proposed (e.g. Refs [2, 5, 4, 6, 8, 11]). A review of surface parameterization methods is beyond the scope of this paper. The reader may consult the recent survey [5] for detailed expositions. In general, the topology of 3D surfaces is always the same as a disk [2, 4]. For complex surfaces, a popular technique is to partition those surfaces into a set of charts corresponding to the simply shapes (e.g. Refs [8, 11]). In especial, if an open connected surface has more than one boundary (i.e. multiple holes), a general solving method is to partition the surface into a set of charts. Texture mapping, which is widely used to enhance the reality of 3D models in computer graphics, is one of important applications on surface parameterization.

However, there are two limitations on texture mapping on complex open connected surfaces based on the partition techniques.

- It is a non-trivial task to build a reasonable partition for texture mapping, and it is highly complicated when dealing with large and complex surfaces.

- Partition often produces high distortion and discontinuity nearby partition boundaries (see Fig. 2(c)).

One possible solution is to fill holes (e.g. [7, 9]) and then use the common parameterization techniques to compute the mapping of surfaces. The method requires the expenditure of large amounts of time and space for large and complex surfaces. Furthermore, it is also an additional computation for parameterizing and changing the topology of original surfaces, so it is not a good choice for surface parameterization.

To solve those problems, we apply a form of Hessian-based Locally Linear Embedding (HLLE) [3]. HLLE is a dimensionality reduction technique that finds a low-dimensional embedding of a set of points lying on an open connected manifold embedded in high-dimensional input space. There have been some studies that extend nonlinear dimensionality reduction methods [10, 12] to surface parameterization in the last few years. Zigelman et al. [14] combine multidimensional scaling (MDS) with the fast marching method that computes a geodesic distance from a given surface point. In essence, their method is an ex-

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tension of Isomap [12] to surface parameterization. Zhou et al. give a similar application using the Isomap method [13]. However, those dimensionality reduction techniques require surfaces to be convexity [3], so they are not suitable for an open connected surface with multiple holes. Our method may be regarded as an extension of the HLLE method. Moreover, most of the previous parameterization methods require fixed boundary conditions (e.g. [4]). In the proposed method, boundary conditions are not required for a valid solution, so our method is also a free-boundary parameterization method [2]. In computer graphics and geometric modeling, shapes are commonly represented by triangular meshes, and surfaces referred to in this paper are triangular meshes.

The remainder of this paper is organized as follows. In Section 2, we briefly review dimensionality reduction techniques. In Section 3, the parameterization based on HLLE is described. In Section 4, the application on texture mapping for an open connected surface is presented. In Section 5, some experimental results are introduced, and conclusions are given in Section 6.

2. Dimensionality reduction

Dimensionality reduction, whose goal is to find a low-dimensional embedding of high-dimensional input data, is an important process in many areas, such as artificial intelligence and data mining [1, 10, 12]. Nonlinear dimensionality reduction, including Isomap [12], locally linear embedding (LLE) [10], and Laplacian Eigenmaps (LEM) [1], currently is an active area of research. Isomap performs dimensionality reduction by applying MDS on the geodesic distance matrix. LLE computes low-dimensional and neighborhood-preserving embedding of high-dimensional inputs. LEM attempts to maintain ordering between points in a local embedding. However, most of those methods assume that the high-dimensional data lies on a convex manifold. Donoho et al. propose the HLLE method [3], which is suitable for an open connected manifold. The HLLE method may be viewed as a modification of LLE. The LLE method solves a linear least-square problem to estimate the local weights [10], while the HLLE method calculates a singular value decomposition and uses Gram-Schmidt orthonormalization to estimate each local Hessian [3]. The major feature is that, HLLE is a global embedding and is locally isometric to an open, connected manifold.

For the HLLE method, the input of dimensionality reduction is a set of high-dimensional points lying on an open connected manifold. In particular, we consider a manifold as a 3D surface. Most previous dimensionality reduction methods [10, 12] are suitable for convex surfaces. However, the assumption is not suitable for surfaces with multiple holes. Here, we use an example, which is provided in Ref. [3], to show the difference of embedding results generated by three methods: Isomap, LLE, and HLLE, as illustrated in Fig. 1. The original data contains a set of 600 points sampled on a 3D surface with one hole. The LLE result is generated using the code published by Roweis et al. [10], and the Isomap embedding is produced using the code published by Tenenbaum et al. [12]. The correct parameter space generated from the original data is a square with a central square removed, while the embedding obtained by the HLLE method is perfect. The detailed expositions of the HLLE algorithm can be found in Donoho et al. [3].

3. Parameterization based on HLLE

The first step of our procedure is to find an appropriate parameterization method without any partition. Our algorithm builds upon the dimensionality reduction method HLLE [3]. Given an open connected surface S with n vertices $\{\mathbf{x}_i \in \mathbb{R}^3 | i = 1, ..., n\}$, we first apply the HLLE method to the 3D mesh surface S. The parameterization method based on HLLE can be described as follows.

1. The first step in HLLE determines each data point's neighborhood with k-neighbors in Euclidean distance. Since S is a 3D mesh surface and its connectivity is well-known, we choose the 1-ring neighborhood of a vertex instead of k-neighbors. For each vertex $\mathbf{x}_i$, let N_i be a set of adjacent vertex indices of $\mathbf{x}_i$. For each neighborhood $N_i, i = 1, ..., n$, form a matrix M^i whose rows consist of the points $\mathbf{x}_j - \mathbf{x}_i, j \in N_i$.

2. Perform a singular value decomposition for each M^i and obtain local coordinates. Use the local coordinates and Gram-Schmidt orthonormalization to estimate each local Hessian.

3. Build Hessian matrix with estimating the local Hessian over neighborhood.

4. Perform an eigenanalysis of Hessian matrix, and compute the three eigenvectors corresponding to three smallest eigenvalues. Discard the eigenvector which is corresponding to zero eigenvalue. The remaining two eigenvectors form the $n \times 2$ matrix denoted by V. Define the $n \times n$ matrix $R = VV^T$, and the $2 \times n$ matrix of embedding coordinates is obtained from $W = V^T R^{-1/2}$, where V^T denotes the transpose matrix of V.

The main difference between parameterization and HLLE is in the first and second step. The last two steps are similar to the HLLE algorithm.

4. Application on texture mapping

The parameterization method presented in this paper provides a powerful toolbox. In this section we consider its

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application on texture mapping.

After applying the HLLE based parameterization to the surface points, we get a 2D mapping of the surface. Thus, the mapping of every point on the surface to its corresponding 2D flattened point is also obtained. Given a flat texture image, we can easily map each point from the 2D mapping to a point on the texture image. Then we will map the texture back to the surface. The technique is straightforward.

The HLLE method is computationally expensive because of the matrix eigenanalysis [3]. In practice, we select a subset of the vertices and apply the HLLE parameterization on this subset. After flattening the subset, we interpolate the local coordinates to find the local mapping of the rest of the vertices. The strategy is also used by Zigelman et al. [14].

5. Experiment results

We demonstrate our results in Figs. 2 and 3. The execution time is reported on a Pentium IV 1.70GHz processor with 256M RAM excluding that of loading meshes. Figs. 2(d) and 3(c) are obtained in about 22 seconds and 7 seconds, respectively. In Fig. 2, we compare our result to the result based on partition of a face model with one hole. Fig. 2(b) shows the new model partitioned by a curve. Observe that highly distorted and discontinuous texture appears nearby the partition boundary in Fig. 2(c), and the texture is smooth by our method in Fig. 2(d). In Fig. 3, we also show a more complex surface with seven holes. As can be seen, the proposed HLLE based parameterization reduces the deformations and better preserves the local and global structure of the texture.

6. Conclusions

In this paper, we have presented a novel mesh parameterization method for an open connected surface based on the HLLE method, and gave its application on texture mapping. The proposed approach has several advantages: 1) Boundary conditions are not required for a surface parameterization. 2) Our method dose not use any partition technique and can preserve the local and global structure of the texture.

The major drawback in our current implementation is that the HLLE mapping method may contain local overlapping triangles in the planar embedding. In order to produce unfolding mapping, a possible solution is to apply the free-boundary method introduced by Desbrun et al. [2], which is an iterative method, on the resulting embedding. In the future, it is an interesting topic to improve the HLLE method for minimizing geometric stretch in some ways.

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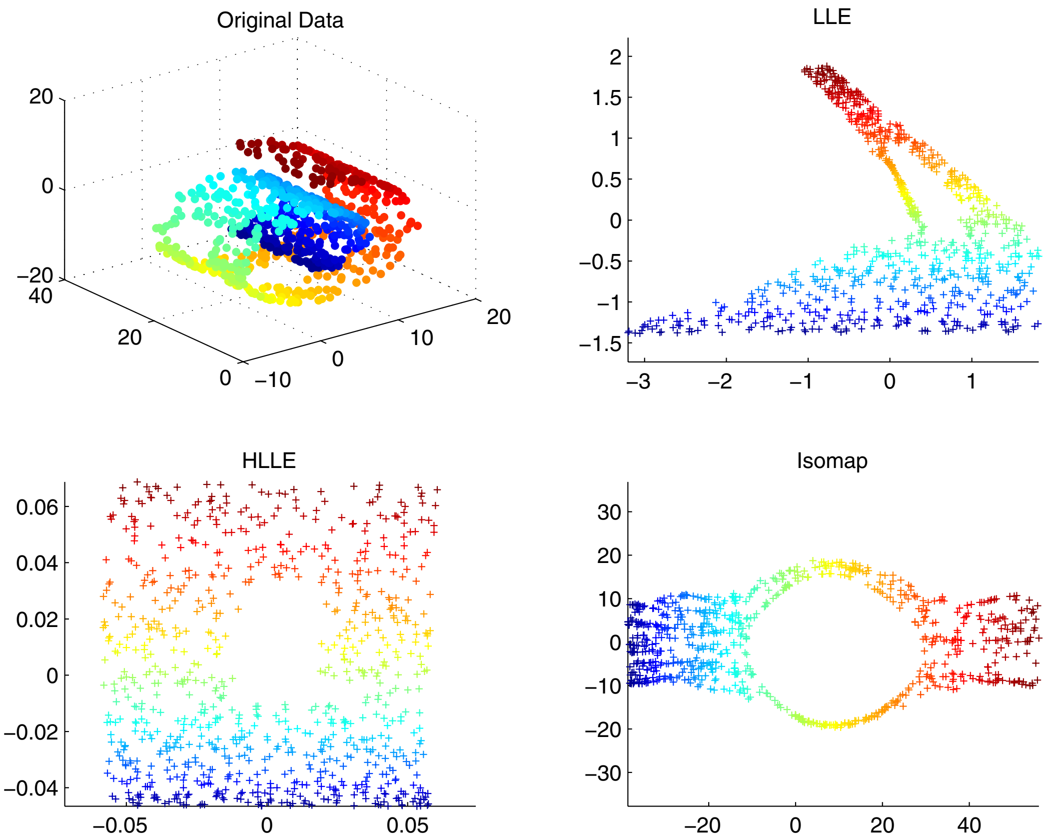
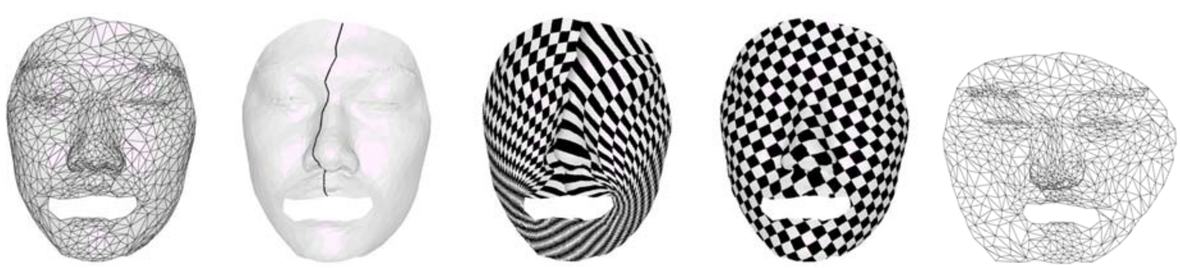


Figure 1: A comparison of embedding results.



(a) (b) (c) (d) (e)

Figure 2: A comparison between partition-based mapping and the HLLE mapping on a face model. (a) The original mesh with 824 vertices. (b) Partition. (c) A checker texture mapped to (b) using Floater's method [4]. (d) The same checker texture mapped to (a) using the HLLE mapping. (e) The corresponding parameterization to (a) using the HLLE mapping.

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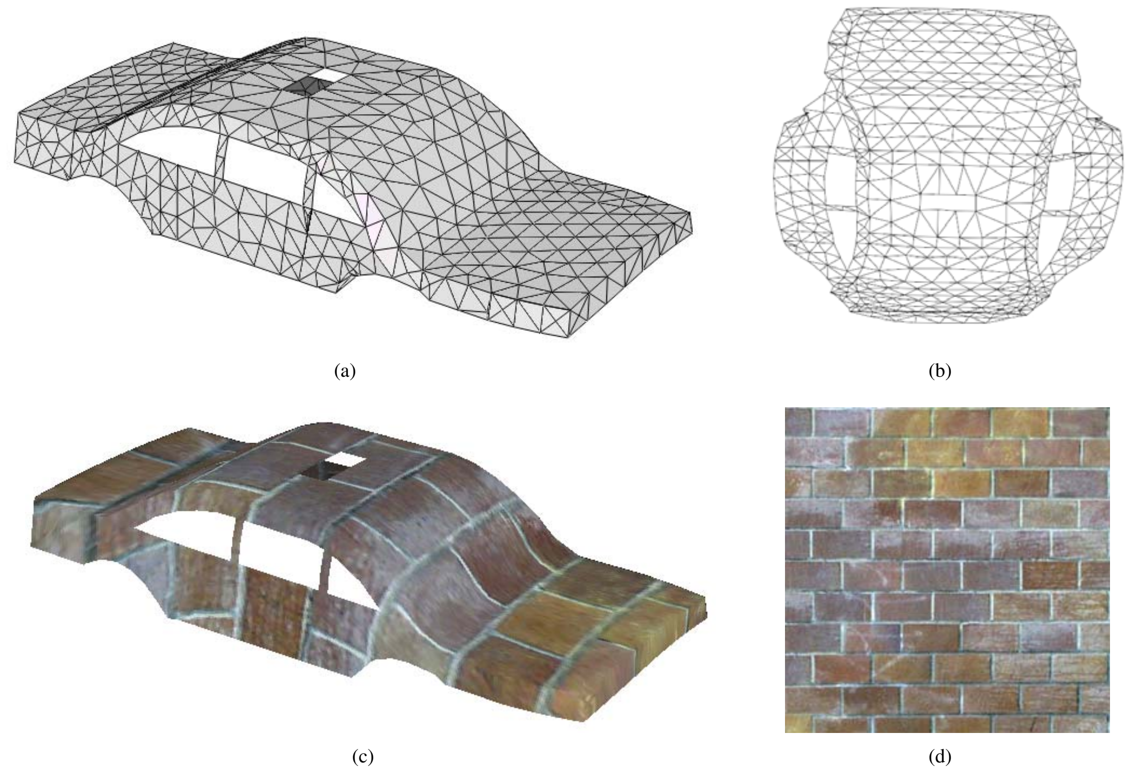

Figure 3: An example of texture mapping on a complex car model. (a) The original mesh with 543 vertices. (b) The corresponding parameterization using the HLLE mapping. (c) Texture mapped to (a). (d) The corresponding texture image.

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